Miss Fannie Hartson, Mexico, New York. By the first marriage, one son was born to them, who died in 1889. The second marriage was blessed with six children, five of whom are living. Professor Wood is a member of the Methodist Church—and, also, a member of the Official Board of the Methodist Church.

NON-EUCLIDEAN GEOMETRY: HISTORICAL AND EXPOSITORY.

By GEORGE BRUCE HALSTED, A. M., (Princeton), Ph. D., (Johns Hopkins), Member of the London Mathematical Society, and Professor of Mathematics in the University of Texas, Austin, Texas.

[Continued from the July-August Number.]

SCHOLION I. *In which is weighed the attempt of Proclus.* After the theorems so far demonstrated by me, independently of the Euclidean postulate, toward an exact demonstration of which they should all conspire; in my judgment it is well if I diligently weigh the labors of certain well-known geometers in the same endeavor.

I begin from Proclus, of whom Clavius in the Elements after P. XXVIII, Book I, gives the following assumption:

If from one point two straight lines making an angle are produced infinitely, their distance will exceed every finite magnitude.

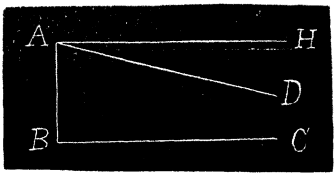

FIG. 20.

But Proclus demonstrates indeed (as Clavius there well remarks) that two straights (fig. 20) as suppose AH, AD going out from the same point A toward the same parts, always diverge the more from each other, the greater the distance from the point A, but not also that this distance increases beyond every finite limit that may be designated, as was requisite for his purpose.

In which place the aforesaid Clavius cites the example of the Conchoid of Nikomedes, which going out from the same point A as the straight AH toward the same parts, so recedes always more from it, that nevertheless only at an infinite production is their distance equal to a certain finite sect AB standing perpendicular to AH and BC produced to infinity toward the same parts.

Why may not the same be said of the two supposed straight lines AH, AD, unless a special reason constrains to the contrary?

Nor here can Clavius be blamed that he opposes to Proklos this property of the Conchoid, which cannot be demonstrated except with the aid of many theorems resting upon the here controverted postulate.

For I say from this itself the force of the Clavian rebuttal is confirmed; for it is certain from this postulate being assumed that truly it follows manifestly, that two lines protracted to infinity, one straight, and the other

curved, can recede one from the other ever more within a certain finite determinate limit; whence at any rate may arise a suspicion lest the same may be able to happen for two straight lines, unless otherwise demonstrated.

But not therefore,after I in the corollary to the preceding proposition I have made manifest the absolute truth of the aforesaid assumption,is it possible immediately to go over to the assertion of the Euclidean postulate. For previously must also be demonstrated, that those two straights AH, BC, which with the transversal AB make two angles toward the same parts equal to two right angles, as for example each a right angle, do not also, protracted toward these parts to infinity, always separate more from one another beyond all finite assignable distance. For if one chooses to presume the affirmative, which is indeed entirely true in the hypothesis of acute angle; it certainly will not be a legitimate consequence, that the straight AD in any way cutting the angle HAB, hence of course making at the same time two internal angles DAB, CBA toward the same parts less than two right angles; that, I say, this straight AD, produced to infinity must at length meet with BC produced; even if it were at another time demonstrated, that the distance of the two AH, AD produced to infinity ever greater goes out beyond all finite limit that may be assigned.

But that the aforesaid Clavius should have judged the truth of this assumption sufficient for demonstrating the postulate here in question; that ought to becondoned because of the opinion preconceived by Clavius about equidistant straight lines, which we may discuss more conveniently in a subsequent Scholion.

INTRODUCTION TO SUBSTITUTION GROUPS.

By G. A. MILLER, Ph. D., Leipzig, Germany.

(Continued from the July-August Number.)

Construction of Intransitive Groups.

Suppose that we have an intransitive group (G) involving the six letters a, b, c, d, e, f and that in this group a is replaced by b, c, and d but not by e or f. This group must have at least one substitution (s_1) in which a is replaced by b, one (s_2) in which a is replaced by c and one (s_3) in which a is replaced by d. In some power of s_1 (which, from the definition of a group, must also be in G) b is replaced by a.* Let this substitution be denoted by s'_1 and consider the following substitution of G:

$$s'_1, \quad s'_1 \; s_2, \quad s'_1 \; s_3.$$

In the first of these b is replaced by a, in the second by c and in the third by d. Hence we see that the hypothesis that a is replaced by each of the

*Suppose s_1 were one of the following substitutions: ab, abc, $abcd$, $ab.cd$; then b would be replaced by a in the first power of the first and last substitutions, in the second power of the second substitution and the third power of the third.